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Geometric Axioms

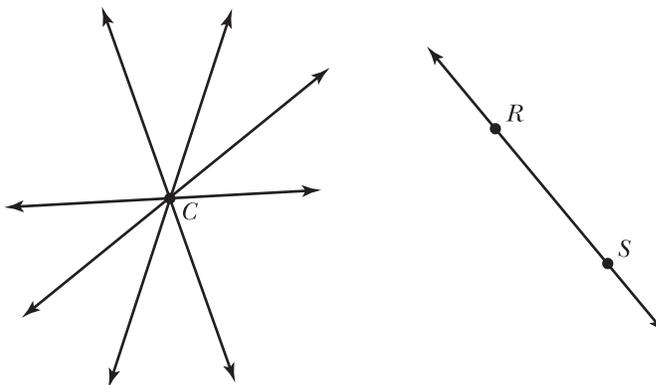
Obvious things that don't need proof

Points are usually indicated by capital letters, such as C , R , and S in this problem.

4.1 Complete the following statement and justify your answer.

Two distinct points define a _____.

Two distinct points define a line. An infinite number of lines can be drawn through the single point C below, but only one line can be drawn connecting points R and S .

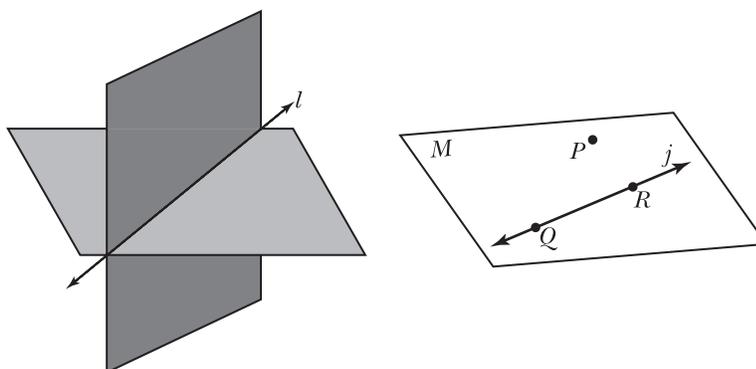


4.2 Complete the following statement and justify your answer.

_____ distinct, _____ points define a plane.

Noncollinear means "not all lying on the same line."

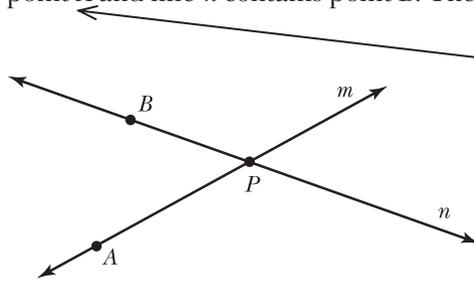
Three distinct, noncollinear points define a plane. According to Problem 4.1, two distinct points are required to define a one-dimensional line. A third point not belonging to that line (and, therefore, noncollinear) is required to define a plane. An infinite number of planes may be drawn through line l below (including the two planes illustrated in the diagram), but only plane M contains points Q and R (on line j) and noncollinear point P .



Note that the surface of plane M extends infinitely, as does line j , which lies on the plane.

4.3 Draw intersecting lines m and n and describe their intersection. Assume that m and n do not describe the same line.

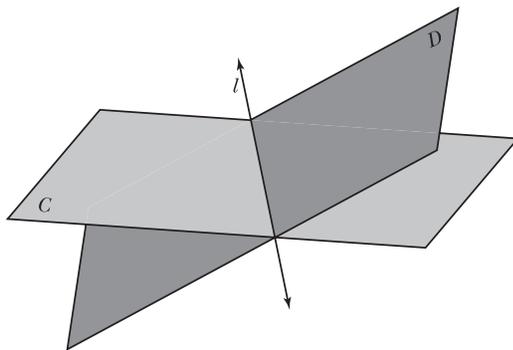
If two lines intersect, they intersect at exactly one point. In the diagram below, line m contains point A and line n contains point B . The lines intersect at point P .



Lines are named either with a lowercase letter (such as m) or by two points on the line (such as \overleftrightarrow{AP}).

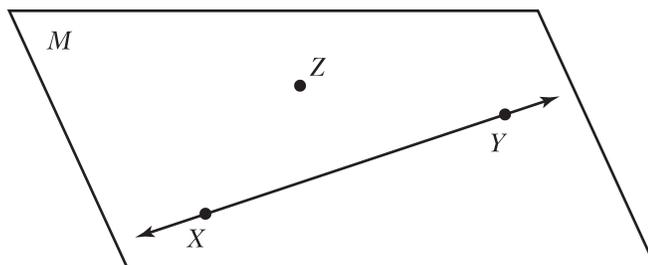
4.4 Draw intersecting planes C and D and describe their intersection.

A line defines the intersection of two planes. Consider the walls of a rectangular room. Each vertical wall meets its adjacent walls at a straight line, the corner of the room. In the following diagram, planes C and D intersect at line l .



4.5 Construct a diagram containing a line \overleftrightarrow{XY} and a noncollinear point Z . How many planes may be drawn that contain both? Justify your answer.

Draw a line passing through points X and Y . As long as both points appear on the line, their placement is irrelevant. Construct a point Z that does not belong to that line. According to the axiom described in Problem 4.2, three distinct and noncollinear points (like X , Y , and Z) define exactly one plane, labeled M in the diagram below.



X and Y ARE collinear, but the line doesn't pass through Z . That means that all three points (X , Y , and Z) are NOT collinear.

Reflections

Using a line as a mirror

21.8 Complete the following statement.

Given $R_l: P \rightarrow P'$, R_l is a reflection in line l if l is the _____ of every segment _____ and $R_l: \underline{P} \rightarrow \underline{P}$ when P lies on l .

Given $R_l: P \rightarrow P'$, R_l is a reflection in line l if l is the perpendicular bisector of every segment $\overline{PP'}$ and $R_l: \underline{P} \rightarrow \underline{P}$ when P lies on l . In other words, if you connect every pre-image to the corresponding image, all the resulting segments have the same perpendicular bisector, line l .

Note: Problems 21.9–21.14 refer to a triangle with vertices $A = (-3, -1)$, $B = (0, 3)$, and $C = (4, 2)$.

21.9 Let $\Delta A'B'C'$ be the reflection of ΔABC across the x -axis. Identify the coordinates of the vertices of $\Delta A'B'C'$.

The x -axis is the horizontal line $y = 0$. According to Problem 21.8, $y = 0$ is the perpendicular bisector of the segments connecting corresponding pre-images and images. Thus, $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ are vertical segments.

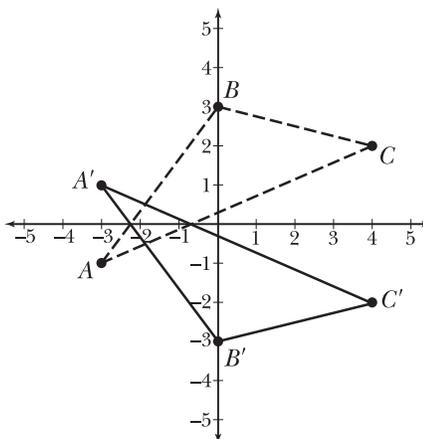
Consider points A and A' . Because the x -axis is the perpendicular bisector of $\overline{AA'}$, the distance from A to the x -axis must be equal to the distance from the x -axis to A' . Thus, A' is 1 unit above the x -axis: $A' = (-3, 1)$.

Similarly, pre-images B and C share x -coordinates with corresponding images B' and C' , and have opposite y -coordinates: $B' = (0, -3)$ and $C' = (4, -2)$.

Note: Problems 21.9–21.14 refer to a triangle with vertices $A = (-3, -1)$, $B = (0, 3)$, and $C = (4, 2)$.

21.10 Let $\Delta A'B'C'$ be the reflection of ΔABC across the x -axis, as described in Problem 21.9. Graph both triangles in the coordinate plane.

Plot and connect the vertices of ΔABC . Graph $\Delta A'B'C'$ by connecting the points identified in Problem 21.9: $A' = (-3, 1)$, $B' = (0, -3)$, and $C' = (4, -2)$.



Unless the pre-image is actually ON line l , in which case transformation R leaves the point alone and doesn't assign it a new set of coordinates.

A and A' are endpoints of a vertical segment, so they have the same x -value ($x = -3$). Their y -values are opposites: A is 1 unit below the x -axis ($y = -1$), so A' is 1 unit above the x -axis ($y = 1$).

Note: Problems 21.9–21.14 refer to a triangle with vertices $A = (-3,-1)$, $B = (0,3)$, and $C = (4,2)$.

21.11 Let $\Delta A'B'C'$ be the reflection of ΔABC across the y -axis. Identify the coordinates of the vertices of $\Delta A'B'C'$.

The y -axis is the perpendicular bisector of all segments connecting the pre-images to the corresponding images. Thus, the vertical line $x = 0$ bisects horizontal segments $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$.

Consider points A and A' . Because the y -axis is the perpendicular bisector of $\overline{AA'}$, the distance from A to the y -axis must be equal to the distance from the y -axis to A' . Thus, A' is 3 units right of the y -axis: $A' = (3,-1)$.

Similarly, pre-images B and C share y -coordinates with corresponding images B' and C' , and have opposite x -coordinates: $B' = (0,3)$ and $C' = (-4,2)$.

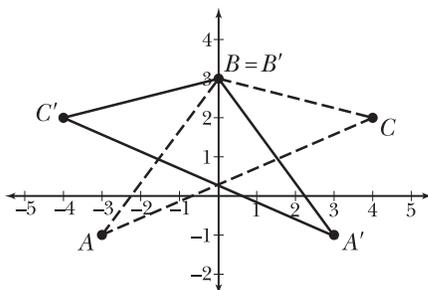
In Problems 21.11–21.12, you're reflecting across the y -axis instead of the x -axis.

B is on the y -axis, the line you're reflecting across. That means the transformation does not affect B . Its coordinates stay the same because the opposite of 0 is 0.

Note: Problems 21.9–21.14 refer to a triangle with vertices $A = (-3,-1)$, $B = (0,3)$, and $C = (4,2)$.

21.12 Let $\Delta A'B'C'$ be the reflection of ΔABC across the y -axis, as described in Problem 21.11. Graph both triangles in the coordinate plane.

Plot and connect the vertices of ΔABC . Graph $\Delta A'B'C'$ by connecting the points identified in Problem 21.11: $A' = (3,-1)$, $B' = (0,3)$, and $C' = (-4,2)$.



Note: Problems 21.9–21.14 refer to a triangle with vertices $A = (-3,-1)$, $B = (0,3)$, and $C = (4,2)$.

21.13 Let $\Delta A'B'C'$ be the reflection of ΔABC across the line $y = x$. Identify the vertices of $\Delta A'B'C'$.

Reflecting coordinates across the line $y = x$ reverses the x - and y -coordinates. In other words, reflecting pre-image (a,b) produces image (b,a) . Thus, $A' = (-1,-3)$, $B' = (3,0)$, and $C' = (2,4)$.

The new y -value is the old x -value (and vice versa).