Contents

Geometric Axioms

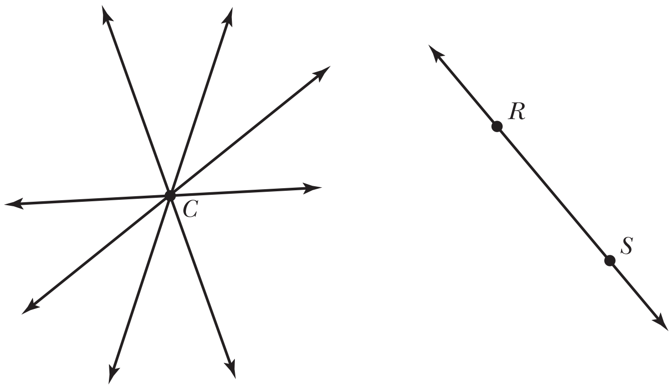
Obvious things that don't need proof

4.1 Complete the following statement and justify your answer.

Two distinct points define a ___________.

Points are usually indicated by capital letters, such as C, R, and S in this problem.

Two distinct points define a line. An infinite number of lines can be drawn through the single point *C* below, but only one line can be drawn connecting points *R* and *S*.

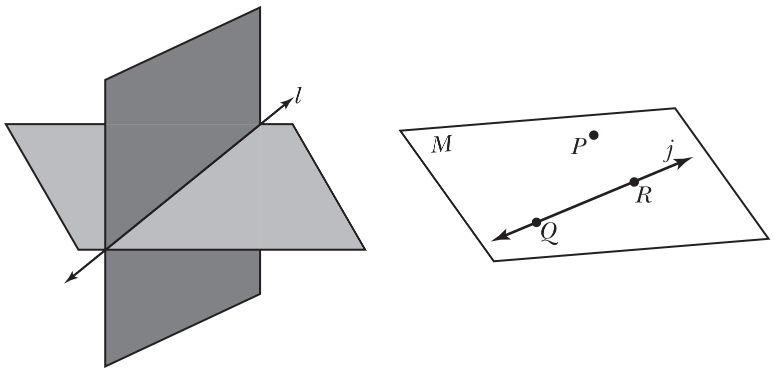


4.2 Complete the following statement and justify your answer.

___________ *distinct,* ___________ *points define a plane.*

Noncollinear means "not all lying on the same line."

Three distinct, noncollinear points define a plane. According to Problem 4.1, two distinct points are required to define a one-dimensional line. A third point not belonging to that line (and, therefore, noncollinear) is required to define a plane. An infinite number of planes may be drawn through line *l* below (including the two planes illustrated in the diagram), but only plane *M* contains points *Q* and *R* (on line *j*) and noncollinear point *P*.

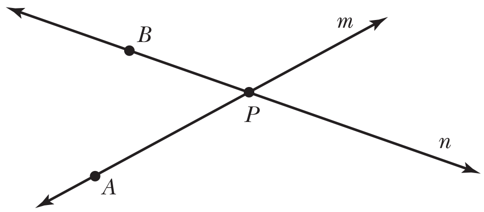


Note that the surface of plane *M* extends infinitely, as does line *j*, which lies on the plane.

4.3 Draw intersecting lines *m* and *n* and describe their intersection. Assume that *m* and *n* do not describe the same line.

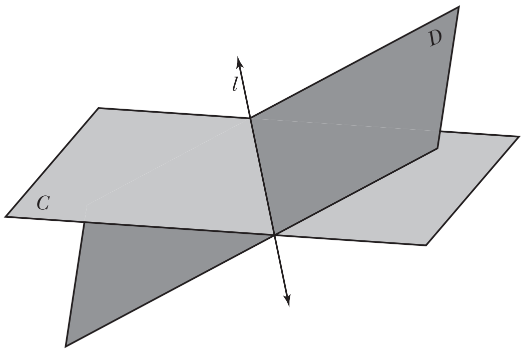
If two lines intersect, they intersect at exactly one point. In the diagram below, line *m* contains point *A* and line *n* contains point *B*. The lines intersect at point *P*.

Lines are named either with a lowercase letter (such as m) or by two points on the line (such as $\overleftrightarrow{AP}$).



4.4 Draw intersecting planes *C* and *D* and describe their intersection.

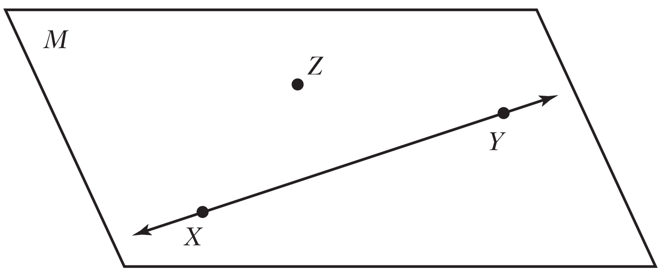
A line defines the intersection of two planes. Consider the walls of a rectangular room. Each vertical wall meets its adjacent walls at a straight line, the corner of the room. In the following diagram, planes *C* and *D* intersect at line *l*.



4.5 Construct a diagram containing a line $\overleftrightarrow{XY}$ and a noncollinear point *Z*. How many planes may be drawn that contain both? Justify your answer.

Draw a line passing through points *X* and *Y*. As long as both points appear on the line, their placement is irrelevant. Construct a point *Z* that does not belong to that line. According to the axiom described in Problem 4.2, three distinct and noncollinear points (like *X*, *Y*, and *Z*) define exactly one plane, labeled *M* in the diagram below.

X and Y ARE collinear, but the line doesn't pass through Z. That means that all three points (X, Y, and Z) are NOT collinear.



Reflections

Using a line as a mirror

21.8 Complete the following statement.

Given R_l:$P \to P'$, Rl is a reflection in line l if l is the ________ of every segment ________ and R_l: _____ $\to$ _____ when P lies on l.

Given R_l:$P \to P'$, Rl is a reflection in line l if l is the perpendicular bisector of every segment $\overline{PP'}$ and R_l: $P \to P$ when P lies on l. In other words, if you connect every pre-image to the corresponding image, all the resulting segments have the same perpendicular bisector, line l.

Unless the pre-image is actually ON line l, in which case transformation R leaves the point alone and doesn't assign it a new set of coordinates.

Note: Problems 21.9–21.14 refer to a triangle with vertices A = (–3,–1), B = (0,3), and C = (4,2).

21.9 Let $\Delta A'B'C'$ be the reflection of ΔABC across the x-axis. Identify the coordinates of the vertices of $\Delta A'B'C'$.

The x-axis is the horizontal line $y = 0$. According to Problem 21.8, $y = 0$ is the perpendicular bisector of the segments connecting corresponding pre-images and images. Thus, $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$ are vertical segments.

Consider points A and A'. Because the x-axis is the perpendicular bisector of $\overline{AA'}$, the distance from A to the x-axis must be equal to the distance from the x-axis to A'. Thus, A' is 1 unit above the x-axis: $A' = (-3,1)$.

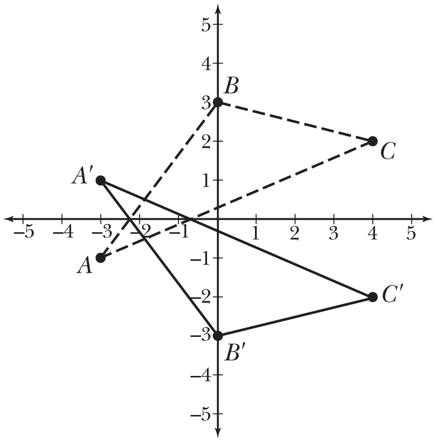
A and A' are endpoints of a vertical segment, so they have the same x-value (x = –3). Their y-values are opposites: A is 1 unit below the x-axis (y = –1), so A' is 1 unit above the x-axis (y = 1).

Similarly, pre-images B and C share x-coordinates with corresponding images B' and C', and have opposite y-coordinates: $B' = (0,-3)$ and $C' = (4,-2)$.

Note: Problems 21.9–21.14 refer to a triangle with vertices A = (–3,–1), B = (0,3), and C = (4,2).

21.10 Let $\Delta A'B'C'$ be the reflection of ΔABC across the x-axis, as described in Problem 21.9. Graph both triangles in the coordinate plane.

Plot and connect the vertices of ΔABC. Graph $\Delta A'B'C'$ by connecting the points identified in Problem 21.9: $A' = (-3,1)$, $B' = (0,-3)$, and $C' = (4,-2)$.



Note: Problems 21.9–21.14 refer to a triangle with vertices A = (–3,–1), B = (0,3), and C = (4,2).

21.11 Let $\Delta A'B'C'$ be the reflection of ΔABC across the y-axis. Identify the coordinates of the vertices of $\Delta A'B'C'$.

> In Problems 21.11–21.12, you're reflecting across the y-axis instead of the x-axis.

The y-axis is the perpendicular bisector of all segments connecting the pre-images to the corresponding images. Thus, the vertical line $x = 0$ bisects horizontal segments $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$.

Consider points A and A'. Because the y-axis is the perpendicular bisector of $\overline{AA'}$, the distance from A to the y-axis must be equal to the distance from the y-axis to A'. Thus, A' is 3 units right of the y-axis: $A' = (3,-1)$.

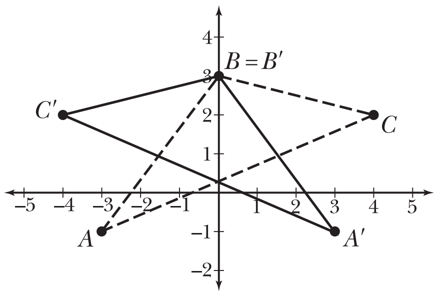
Similarly, pre-images B and C share y-coordinates with corresponding images B' and C', and have opposite x-coordinates: $B' = (0,3)$ and $C' = (-4,2)$.

> B is on the y-axis, the line you're reflecting across. That means the transformation does not affect B. Its coordinates stay the same because the opposite of 0 is 0.

Note: Problems 21.9–21.14 refer to a triangle with vertices A = (–3,–1), B = (0,3), and C = (4,2).

21.12 Let $\Delta A'B'C'$ be the reflection of ΔABC across the y-axis, as described in Problem 21.11. Graph both triangles in the coordinate plane.

Plot and connect the vertices of ΔABC. Graph $\Delta A'B'C'$ by connecting the points identified in Problem 21.11: $A' = (3,-1)$, $B' = (0,3)$, and $C' = (-4,2)$.



Note: Problems 21.9–21.14 refer to a triangle with vertices A = (–3,–1), B = (0,3), and C = (4,2).

21.13 Let $\Delta A'B'C'$ be the reflection of ΔABC across the line $y = x$. Identify the vertices of $\Delta A'B'C'$.

Reflecting coordinates across the line $y = x$ reverses the x- and y-coordinates. In other words, reflecting pre-image (a,b) produces image (b,a). Thus, $A' = (-1,-3)$, $B' = (3,0)$, and $C' = (2,4)$.

> The new y-value is the old x-value (and vice versa).