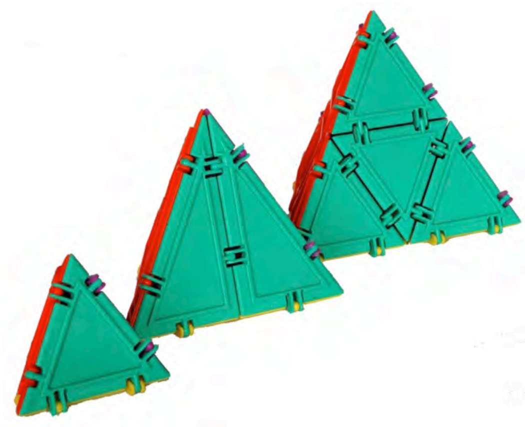
Sample lesson

Scaling, Area, and Volume

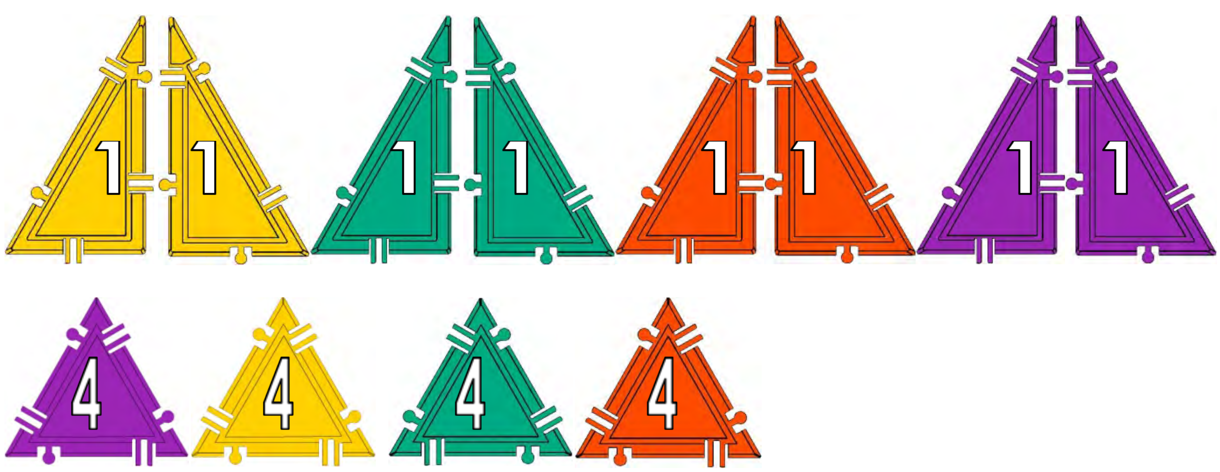


Objective: Learn how scaling lengths affects areas and volumes

Grades: 6-8

Common Core State Standards: 6.G.A.1, 6.G.A.2,6.RP.A.1, 7.G.B.6, 7.RP.A.2, 8.G.A.1, 8.G.A.4

Pieces needed per group of 2-3 students:

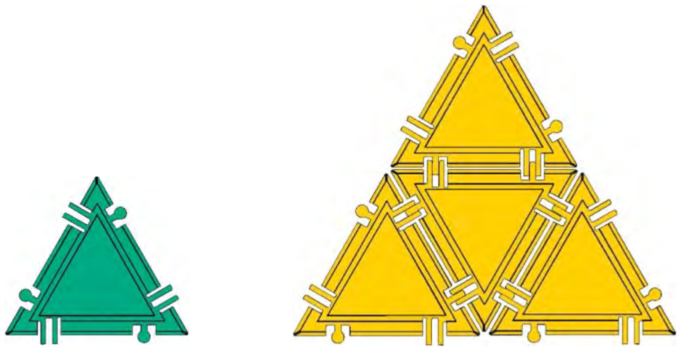


Teacher's prompts to students are in **bold**.

Question 1

Suppose we double the length of each side of an equilateral triangle by 2. By what factor will the area increase?

Answer:

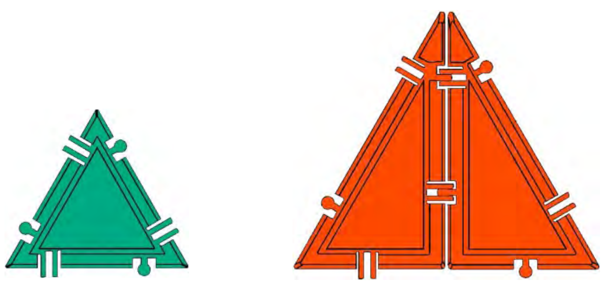


We can simply count the number of small triangles in the large yellow triangle on the right. There are 4 of them. So when we double the length, the area quadruples.

Question 2

What if we increase the length of an equilateral triangle by a factor other than 2? What will the ratio of the areas be in that case?

Have students look at the following equilateral triangles:



Use a ruler to find their approximate side lengths. What is the ratio of the length of the larger one to the length of the smaller one? Answer: about 1.6

What do you expect the ratio of their areas to be? Here students will either take a guess based on "eyeballing" the sizes or compute $(1.6)^2=2.56$.

Compute the areas of both triangles and calculate the ratio of the areas. Does your answer agree with your guess?

Answer: We will use the formula for the area of a triangle $A = \frac{1}{2}\times b\times h$. The area of the larger triangle is about $\frac{1}{2}\times 9.6\times 8.2 cm^2 \approx 39 cm^2$, and the area of the smaller rectangle is $\frac{1}{2}\times 5.9\times 5 cm^2 \approx 15 cm^2$. So the ratio of the areas is about 39/15=2.6.

This is what we would expect. When we doubled the length in the previous question, the area increased by a factor of $2^2 = 4$. When length gets multiplied by 1.6, area increases by a factor of $1.6^2 = 2.56 \approx 2.6$. In general, if the length gets multiplied by a number, the area gets multiplied by (number)^2.

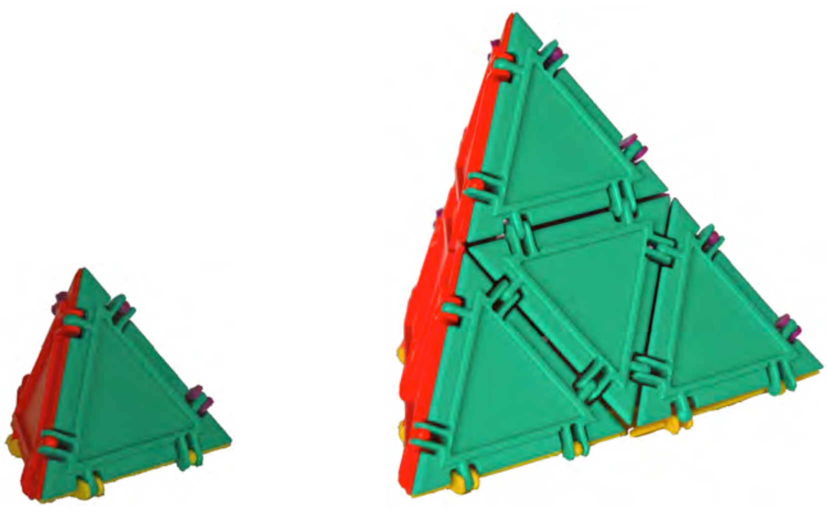
You can also see this from the area equations. Since the larger triangle has all its dimensions equal to the dimensions of the small triangle multiplied by 1.6, we can express its area as

Area of larger triangle=$\frac{1}{2}\times 5.9(1.6)\times 5(1.6)cm^2 = (1.6)^2\times\left(\frac{1}{2}\times 5.9\times 5\right)cm^2 = (1.6)^2\times$area of smaller triangle.

Question 3

What do you think will happen to the volume of a triangular pyramid (tetrahedron) if we increase double the length of each side?

Have students make these two tetrahedra and guess the ratio of their volumes:



After students had their turns guessing ratios, you can have them estimate the ratio experimentally by seeing how many small tetrahedra worth of rice (or any other dry quantity that can be easily measured) will fit into the larger tetrahedron:

The theoretical answer is 8. Students may get somewhat different answers depending on the size of the grains they use and other experimental variables.

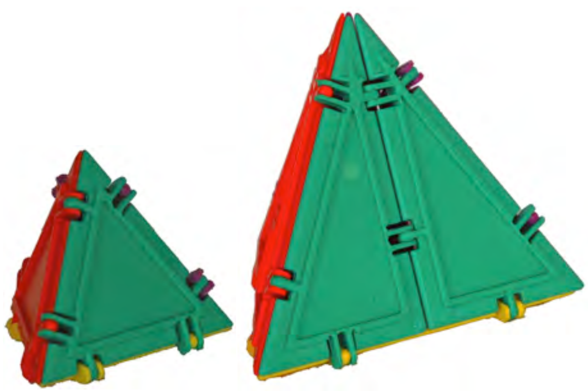
For an extra challenge, covered in a different lesson plan, students can use a geometric construction approach to see why the volume of the larger tetrahedron is exactly 8 times the volume of the smaller one.

Why 8? When we double all the lengths, the height of the tetrahedron also doubles, and the area of the base gets multiplied by $4=2^2$, as we saw in Question 1. Since the formula for the volume of a tetrahedron is $V = \frac{1}{3}\times b\times h$, when we double the height and quadruple the base the new volume becomes $\frac{1}{3}\times(2^2 b)\times(2h) = 2^3\times\frac{1}{3}\times b\times h = 8\times V$.

Question 4

What if we increase the length of an equilateral triangle by a factor other than 2? What will the ratio of the areas be in that case?

Have students consider these two tetrahedra:



We already know from the previous question that the ratio of the side lengths is 1.6. So we will expect the ratio of the volumes to be $(1.6)^3 \approx 4$. Students can confirm this by calculating the volume of each tetrahedron and/or performing the rice experiment.

